

(1) 곱셈공식1

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

예제1

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2, \quad \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2, \quad x \cdot \frac{1}{x} = 1$$

$$(x+x^{-1})^2 = x^2 + x^{-2} + 2, \quad (x-x^{-1})^2 = x^2 + x^{-2} - 2$$

$$(a^x + a^{-x})^2 = a^{2x} + a^{-2x} + 2 \quad (a^x - a^{-x})^2 = a^{2x} + a^{-2x} - 2$$

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2 \quad \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$$

(2) 곱셈공식2

$$(a+b)(a-b) = a^2 - b^2$$

$$(a^2 + b^2)(a^2 - b^2) = a^4 - b^4$$

$$(a-b)(a+b)(a^2 + b^2)(a^4 + b^4) = a^8 - b^8$$

(3) 곱셈공식3

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

예제2

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x \frac{1}{x} \left(x + \frac{1}{x}\right)$$

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3 = x \sqrt{x} + \frac{1}{x \sqrt{x}} + 3 \sqrt{x} \frac{1}{\sqrt{x}} \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$$

$$(a^x + a^{-x})^3 = a^{3x} + a^{-3x} + 3a^x a^{-x} (a^x + a^{-x})$$

$$(x+x^{-1})^3 = x^3 + x^{-3} + 3x x^{-1} (x+x^{-1})$$

예제3

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x \frac{1}{x} \left(x - \frac{1}{x}\right)$$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^3 = x \sqrt{x} - \frac{1}{x \sqrt{x}} - 3 \sqrt{x} \frac{1}{\sqrt{x}} \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$$

$$(a^x - a^{-x})^3 = a^{3x} - a^{-3x} - 3a^x a^{-x} (a^x - a^{-x})$$

$$(x+x^{-1})^3 = x^3 + x^{-3} + 3x x^{-1} (x+x^{-1})$$

(4) 곱셈공식4

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$$

$$(x-a)(x-b) = x^2 - (a+b)x + ab$$

$$(x-a)(x-b)(x-c) = x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc$$

예제4

$$(\alpha+1)(\beta+1)(\gamma+1) = 1^3 + (\alpha+\beta+\gamma)1^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)1 + \alpha\beta\gamma$$

$$(2-\alpha)(2-\beta)(2-\gamma) = 2^3 - (\alpha+\beta+\gamma)2^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)2 - \alpha\beta\gamma$$

(5) 식의 변형

$$(x+y)^2 = (x-y)^2 + 4xy \quad \left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4x \frac{1}{x}$$

$$(x-y)^2 = (x+y)^2 - 4xy \quad \left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4x \frac{1}{x}$$

예제5

$$x^4 + y^4 = (x^2 + y^2)^2 - 2x^2y^2$$

$$x^5 + y^5 = (x^2 + y^2)(x^3 + y^3) - x^2y^2(x+y)$$

$$x^6 + y^6 = (x^3 + y^3)^2 - 2x^3y^3$$

$$x^7 + y^7 = (x^3 + y^3)(x^4 + y^4) - x^3y^3(x+y)$$

$$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$$