

일차방정식의 해법

$$ax = b \dots a \neq 0 : x = \frac{b}{a}$$

$$a = 0 : b = 0, \text{ 부정 (many)}$$

$$a = 0 : b \neq 0, \text{ 불능 } (\emptyset)$$

예제1

$(a^2 - 1)x = 1 - a$ 가 근을 갖지 않도록 $a = ?$

$$\Rightarrow (a+1)(a-1)x = 1 - a = -(a-1) \quad \therefore a = -1$$

$$\ast \sqrt{x^2} = |x| \begin{cases} x \geq 0 : x \\ x < 0 : -x \end{cases}$$

$$|x-1| \begin{cases} x \geq 1 : x-1 \\ x < 1 : -x+1 \end{cases}$$

예제2

$|x-1| = 2x+7$ 을 풀어라.

$$|x-2| \begin{cases} x \geq 2 : x-2 \\ x < 2 : -x+2 \end{cases}$$

$$|x+1| \begin{cases} x \geq -1 : x+1 \\ x < -1 : -x-1 \end{cases}$$

$$\Rightarrow \text{i) } x \geq 1 : x-1 = 2x+7, -x=8 \dots x=-8 (\times)$$

$$\text{ii) } x < 1 : -x+1 = 2x+7,$$

$$-3x=6 \dots x=-2 (\bigcirc)$$

$$\therefore x = -2$$

예제3

$|x-1| = |3-x|$ 을 풀어라.

$$\Rightarrow x-1 = \pm(3-x)$$

$$\text{i) } x-1 = 3-x, 2x=4, x=2$$

$$\text{ii) } x-1 = -(3-x), x-1 = -3+x, -1 \neq -3$$

$$\ast |x| = 3 \dots x = \pm 3$$

$$\ast x^2 = y^2, x^2 - y^2 = 0$$

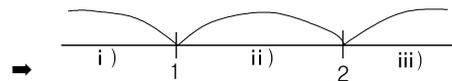
$$|x| = |y| \dots x = \pm y \quad (x+y)(x-y) = 0$$

$$\ast |x| = a \dots x = \pm a \quad (a > 0)$$

$$|A| = |B| \dots A = \pm B$$

예제4

$|x-1| + |x-2| = x+3$ 을 풀어라.



$$\text{i) } x < 1 : -x+1 - x+2 = x+3$$

$$0 = 3x, x = 0$$

$$\text{ii) } 1 \leq x < 2 : x-1 - x+2 = x+3$$

$$-x+1 = 3 \dots x = -2 (\times)$$

$$\text{iii) } x \geq 2 : x-1 + x-2 = x+3$$

$$x-3 = 3 \dots x = 6$$

$$\therefore \text{ i) ii) iii) 의해 } x = 0, 6$$

$$\ast |x| = 3 \dots x = \pm 3$$

$$|x| = |y| \dots x = \pm y$$

$$|x| = a \dots x = \pm a \quad (a > 0)$$

$$A^2 = B^2 \dots A = \pm B$$

$$(x-1)^2 = (x+3)^2 \Rightarrow x-1 = \pm(x+3)$$

$$(x^2 - 2x + 3)^2 = (2x-1)^2 \Rightarrow x^2 - 2x + 3 = 2x - 1$$