

예제1

$$\begin{aligned} & \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{x-2} = \lim_{x \rightarrow 2} (x+2) = 4 \end{aligned}$$

예제2

$$\lim_{x \rightarrow 1} \frac{x^3 - x^2 - 4x + 4}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2 - 4)}{(x+1)(x-1)} = \frac{-3}{2}$$

예제3

$$\begin{aligned} & \lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3} = \lim_{x \rightarrow 9} \frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)} \\ &= 3+3=6 \end{aligned}$$

예제4

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{\sqrt{2}x} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{2+x} - \sqrt{2})(\sqrt{2+x} + \sqrt{2})}{\sqrt{2}x(\sqrt{2+x} + \sqrt{2})} \\ &= \lim_{x \rightarrow 0} \frac{x+2-2}{\sqrt{2}x(\sqrt{2+x} + \sqrt{2})} = \frac{1}{4} \end{aligned}$$

예제5

$$\begin{aligned} & \lim_{x \rightarrow -8} \frac{x+8}{\sqrt[3]{x}+2} \\ &= \lim_{x \rightarrow -8} \frac{(x+8)(\sqrt[3]{x^2} - 2\sqrt[3]{x} + 4)}{x+2^3} \\ &= \sqrt[3]{64} - 2\sqrt[3]{-8} + 4 = 12 \end{aligned}$$

예제6

$$\lim_{x \rightarrow -\infty} \frac{2x}{\sqrt{x^2+3}-4} = -2$$

예제7

$$\lim_{x \rightarrow 0} \frac{1}{x} \left(1 + \frac{1}{x-1} \right) = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{x}{x-1} = \frac{1}{-1} = -1$$

예제8

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sqrt{x+1}} - 1 \right) \\ &= \lim_{x \rightarrow 0} \frac{1}{x} \frac{1 - \sqrt{x+1}}{\sqrt{x+1}} = \lim_{x \rightarrow 0} \frac{1}{x} \frac{1 - (x+1)}{\sqrt{x+1}(1 + \sqrt{x+1})} \\ &= \lim_{x \rightarrow 0} \frac{1}{x} \frac{-x}{\sqrt{x+1}(1 + \sqrt{x+1})} = -\frac{1}{2} \end{aligned}$$

예제9

$$\begin{aligned} & \Rightarrow \lim_{x \rightarrow 1} \frac{ax - 2x^2}{x-1} = b \text{ ... 분자 : } a-2=0, a=2 \\ & ax - 2x^2 = 2x - 2x^2 = 2x(1-x) \\ & \therefore \lim_{x \rightarrow 1} \frac{2x(x-1)}{x-1} = \lim_{x \rightarrow 1} -2x = -2 = b \end{aligned}$$

예제10

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{x^2 + ax + b}{x-1} = 4 \\ & \Rightarrow \text{분자 : } 1+a+b=0, b=-a-1 \\ & \lim_{x \rightarrow 1} \frac{x^2 + ax - a - 1}{x-1} = \lim_{x \rightarrow 1} \frac{(x-1)(a+x+1)}{x-1} = a+2 = 4 \\ & \therefore a=2, b=-3 \end{aligned}$$

예제11

$\lim_{x \rightarrow \infty} \frac{f(x)}{2x^2 + x + 1} = 1, \lim_{x \rightarrow 2} \frac{f(x)}{x^2 - x - 2} = 1$ 에서 $f(x)$ 를 구하라.

$$\begin{aligned} & \Rightarrow f(x) = 2x^2 + \dots \\ & f(x) = (x-2)(2x+a) \\ & \therefore \lim_{x \rightarrow 2} \frac{(x-2)(2x+a)}{(x-2)(x+1)} = \frac{4+a}{3} = 1 \dots a = -1 \\ & \therefore f(x) = (x-2)(2x-1) = 2x^2 - 5x + 2 \end{aligned}$$