

여러 가지 로그의 계산

예제1

$$\begin{aligned} & 2\log_3 \sqrt{3} + 3\log_3 2 + 6\log_3 \frac{\sqrt{2}}{2} \\ &= \log_3 \sqrt{3}^2 + \log_3 2^3 + \log_3 \left(\frac{\sqrt{2}}{2} \right)^6 \\ &= \log_3 3 + \log_3 8 + \log_3 \frac{1}{8} = \log_3 (3 \cdot 8 \cdot \frac{1}{8}) = \log_3 3 = 1 \end{aligned}$$

예제2

$$\begin{aligned} & 2\log \frac{5}{3} - \log \frac{7}{4} + 2\log 3 + \frac{1}{2}\log 49 \\ &= \log \frac{25}{9} + \log \frac{4}{7} + \log 9 + \log 7 \\ &= \log \frac{25}{9} \cdot \frac{4}{7} \cdot 9 \cdot 7 = \log 100 = 2 \end{aligned}$$

예제3

$$\begin{aligned} & 3^{2\log_3 4 + \log_3 5 - 3\log_3 2} \\ &= 3^{\log_3 16 + \log_3 5 - \log_3 8} = 3^{\log_3 (16 \cdot 5 \cdot \frac{1}{8})} = 3^{\log_3 10} = 10 \end{aligned}$$

예제4

$$a^{\frac{\log(\log a)}{\log a}} \quad (a > 1) \quad : \quad a^{\log_a(\log a)} = \log a$$

예제5

$$\begin{aligned} & 45^x = 27, \quad 5^y = 81 \quad \text{일 때} \quad \frac{3}{x} - \frac{4}{y} = ? \\ & \Rightarrow x = \log_{45} 27 = 3\log_{45} 3 \\ & \Rightarrow 45^x = 27 \dots 45 = 27^{\frac{1}{x}} = 3^{\frac{3}{x}} \\ & y = \log_5 81 = \log_5 3^4 = 4\log_5 3 \\ & 5^y = 81 \dots 5 = 81^{\frac{1}{y}} = 3^{\frac{4}{y}} \\ & \therefore \frac{3}{x} - \frac{4}{y} = \frac{3}{3\log_{45} 3} - \frac{4}{4\log_5 3} \end{aligned}$$

예제6

$$\begin{aligned} 9 &= 3^{\frac{3}{x}} = \frac{4}{y} \\ &\Rightarrow \frac{1}{\log_{45} 3} - \frac{1}{\log_5 3} \\ &= \log_3 45 - \log_3 5 = \log_3 9 = 2 \quad \therefore 2 \end{aligned}$$

예제7

$$\begin{aligned} 10^x &= a, \quad 10^y = b, \quad 10^z = c \quad (xyz \neq 0) \quad \text{일 때} \quad \log_{ab} \sqrt{b^2 c} = ? \\ &\Rightarrow \log_{10^x \cdot 10^y} \sqrt{10^{2y} \cdot 10^z} \\ &= \log_{10^{x+y}} (10^{2y+z})^{\frac{1}{2}} = \frac{1}{2} \frac{2y+z}{x+y} \log_{10} 10 \end{aligned}$$

예제8

$$\log_2 3 = a, \quad \log_3 7 = b \quad \text{일 때} \quad \log_{42} 56 = ?$$

$$\Rightarrow \log_3 2 = \frac{1}{a},$$

$$\begin{aligned} \log_{42} 56 &= \frac{\log_3 56}{\log_3 42} = \frac{\log_3 2^3 \cdot 7}{\log_3 2 \cdot 3 \cdot 7} = \frac{3\log_3 2 + \log_3 7}{\log_3 2 + \log_3 3 + \log_3 7} \\ &= \frac{\frac{3}{a} + b}{\frac{1}{a} + 1 + b} = \frac{3 + ab}{1 + a + ab} \end{aligned}$$

예제9

$$x = \log_2 \sqrt{3 - 2\sqrt{2}} \quad \text{일 때} \quad 2^x + 2^{-x} = ?$$

$$\begin{aligned} & \Rightarrow 2^x = \sqrt{3 - 2\sqrt{2}} = \sqrt{2 - 1} \\ & \therefore 2^x + 2^{-x} = \sqrt{2} - 1 + \frac{1}{\sqrt{2} + 1} \\ & \quad = \sqrt{2} - 1 + \sqrt{2} + 1 \\ & \quad = 2\sqrt{2} \end{aligned}$$