예제1

$f(x)=x^4-2x^3+x+4$ 일 때

$\lim_{h\to0}\frac{f(1+h)-f(1-h)}{h}=?$

➡ $\lim_{h\to0}\frac{f'(1+h)+f'(1-h)}{1}=2f'(1)=-2$

$f'(x)=4x^3-6x^2+1$, $f'(1)=4-6+1=-1$

예제2

$\lim_{x\to1}\frac{x^{10}+2x-3}{x-1}=\lim_{x\to1}\frac{10x^9+2}{1}=12$

예제3

$f(x)=\begin{cases}3x^2+1 & (x\geqq1)\\ ax+b & (x<1)\end{cases}$ 가 $x=1$ 에 미분가능 할 때,

$a,\ b=?$

➡ i) $f(1)=4=a+b$

ii) $f'(x)=\begin{cases}6x & (x\geqq1)\\ a & (x<1)\end{cases}$ ... $6=a,\ b=-2$

예제4

x^3+ax^2+b 가 $(x-2)^2$ 으로 나눌 때 나눠

떨어지도록 $a,\ b=?$

➡ $x^3+ax^2+b=(x-2)^2Q(x)$

i) $x=2$: $8+4a+b=0$

ii) 미분 : $3x^2+2ax=2(x-2)Q(x)+(x-2)^2Q'(x)$

예제5

$f'(0)=\frac{1}{2}$ 일 때 $\lim_{n\to\infty}n^3\left\{f\left(\frac{2}{n}\right)-f(0)\right\}^3=?$

➡ $\lim_{n\to\infty}\frac{\left\{f\left(\frac{2}{n}\right)-f(0)\right\}^3}{\frac{1}{n^3}}$

$=\lim_{n\to\infty}\left\{\frac{f\left(\frac{2}{n}\right)-f(0)}{\frac{1}{n}}\right\}^3=\lim_{t\to0}\left\{\frac{f(2t)-f(0)}{t}\right\}^3$

$=\lim_{t\to0}\left\{\frac{f(2t)-f(0)}{2t-0}\cdot2\right\}^3=\{f'(0)\cdot2\}^3=1^3=1$

예제6

$f(1)=1,\quad f'(1)=3\quad \lim_{x\to1}\frac{\{f(x)\}^2-1}{x-1}=?$

$\lim_{x\to1}\frac{xf(x)-1}{x^2-1}=?$

(1) $\lim_{x\to1}\frac{\{f(x)\}^2-1}{x-1}=\lim_{x\to1}\frac{2f(x)\cdot f'(x)}{1}$

$=2f(1)f'(1)=2\cdot1\cdot3=6$

(2) $\lim_{x\to1}\frac{xf(x)-1}{x^2-1}=\lim_{x\to1}\frac{f(x)+xf'(x)}{2x}$

$=\frac{f(1)+f'(1)}{2}=\frac{1+3}{2}=2$

예제7

$f'(a)=3$ 일 때 $\lim_{n\to\infty}n\left\{f\left(a+\frac{2}{n}\right)-f\left(a-\frac{2}{n}\right)\right\}=?$

➡ $\lim_{n\to\infty}\frac{f\left(a+\frac{2}{n}\right)-f\left(a-\frac{2}{n}\right)}{\frac{1}{n}}$

$=\lim_{t\to0}\frac{f(a+2t)-f(a-2t)}{t}$

$=\lim_{t\to0}\{f'(a+2t)\cdot2-f'(a-2t)(-2)\}=2f'(a)+2f'(a)$

$=4f'(a)=4\times3=12$